NAG Toolbox for MATLAB

f04ae

1 Purpose

f04ae calculates the accurate solution of a set of real linear equations with multiple right-hand sides using an LU factorization with partial pivoting, and iterative refinement.

2 Syntax

```
[c, aa, bb, ifail] = f04ae(a, b, 'n', n, 'm', m)
```

3 Description

Given a set of real linear equations $AX = B$, the function first computes an LU factorization of A with partial pivoting, $PA = LU$, where P is a permutation matrix, L is lower triangular and U is unit upper triangular. An approximation to X is found by forward and backward substitution. The residual matrix $R = B - AX$ is then calculated using ***additional precision***, and a correction D to X is found by solving $LUD = PR$. X is replaced by $X + D$ and this iterative refinement of the solution is repeated until full machine accuracy has been obtained.

4 References

Wilkinson J H and Reinsch C 1971 *Handbook for Automatic Computation II, Linear Algebra* Springer–Verlag

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda**,$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n matrix A.

2: **b**(**ldb**,$*$) **– double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{m})$

The n by m right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **m – int32 scalar**

Default: The second dimension of the array **b**.

m, the number of right-hand sides.

Constraint: $\mathbf{m} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, ldc, wkspce, ldaa, ldbb

5.4 Output Parameters

1: **c**(**ldc**,$*$) – **double array**

The first dimension of the array **c** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{m})$

The n by m solution matrix X.

2: **aa**(**ldaa**,$*$) – **double array**

The first dimension of the array **aa** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The triangular factors L and U, except that the unit diagonal elements of U are not stored.

3: **bb**(**ldbb**,$*$) – **double array**

The first dimension of the array **bb** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{m})$

The final n by m residual matrix $R = B - AX$.

4: **ifail** – **int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

The matrix A is singular, possibly due to rounding errors.

ifail $= 2$

Iterative refinement fails to improve the solution, i.e., the matrix A is too ill-conditioned.

ifail $= 3$

On entry, $\mathbf{n} < 0$,
or $\mathbf{m} < 0$,
or $\mathbf{lda} < \max(1, \mathbf{n})$,
or $\mathbf{ldb} < \max(1, \mathbf{n})$,
or $\mathbf{ldc} < \max(1, \mathbf{n})$,
or $\mathbf{ldaa} < \max(1, \mathbf{n})$,
or $\mathbf{ldbb} < \max(1, \mathbf{n})$.

7 Accuracy

The computed solutions should be correct to full machine accuracy. For a detailed error analysis see page 107 of Wilkinson and Reinsch 1971.

8 Further Comments

The time taken by f04ae is approximately proportional to n^3.

If there is only one right-hand side, it is simpler to use f04at.

9 Example

```
a = [33, 16, 72;
     -24, -10, -57;
     -8, -4, -17];
b = [-359;
     281;
     85];
[c, aa, bb, ifail] = f04ae(a, b)
```

```
c =
     1
    -2
    -5
aa =
   -8.0000    0.5000    2.1250
  -24.0000    2.0000   -3.0000
   33.0000   -0.5000    0.3750
bb =
     0
     0
     0
ifail =
           0
```